GEOPHYSICAL RESEARCH LETTERS, VOL. 26, NO. 9, PAGES 1191-1194, MAY 1, 1999

The Use of Laser Altimetry in the Orbit and Attitude Determination of Mars Global Surveyor

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Abstract. Altimetry from the Mars Observer Laser Altimeter (MOLA), an instrument on board the Mars Global Surveyor (MGS) spacecraft, has been analyzed for the period of the MGS Science Phasing Orbit-1 (SPO-1) mission phase. Altimeter ranges have been used to improve significantly the orbit and attitude knowledge of the spacecraft by the use of crossover constraint equations derived from short passes of the MOLA data. These constraint equations differ from traditional crossover constraints and exploit the small footprint associated with laser altimetry. The rationale for using this technique with laser altimetry over sloping terrain is laid out and evidence of the resulting benefit is presented.

Introduction: Radar vs. Laser Altimetry in Orbit and Attitude Solutions

Altimetry in the form of "crossovers" is commonly used for satellite orbit determination, but until now applications have been restricted mainly to Earth-orbiting radar altimetric systems. Using altimetry in this manner requires the least knowledge about the surface at which the altimeter is pointed. *Shum et al.* [1990] provide a detailed description of the use of altimeter crossover constraint equations in orbit determination.

Typically, a priori crossover discrepancies have not been sensitive to horizontal orbit error. That is because most altimetry has been obtained with (large footprint) radar instruments over relatively flat surfaces (ice sheets or oceans) from satellites in circular orbits. So, crossover constraint equations have usually been formulated in terms of height discrepancies at points where (horizontal) ground tracks intersect. These horizontal crossover locations are found from orbit solutions before the altimetry is introduced and the crossover constraint equations are unaffected if there is some horizontal error in the a priori orbits.

Crossover constraint equations in the form of height discrepancies directly affect only the radial component of an orbit. However, the cross track and especially the along track components are indirectly affected through the dynamics of the orbit. In other words, a change in the radial component of the orbit will affect the other two components (especially the along track and to a lesser extent the cross track). For this reason it is sometimes necessary to recompute the latitudes and longitudes of the crossovers as the orbit solution is iterated.

For some time now, our group has been modifying our altimetry processing algorithms to exploit the possibilities of laser altimetry [*Zuber et al.,* 1992]. A major consideration is the increased sensitivity to along-track orbit error of laser altimetry over sloping terrain, given that the laser footprints are small relative to the spacing of the observations. Altimetry from the MOLA instrument [*Zuber et al.,* 1992; *Afzal,* 1994] on MGS during the first Science Phasing Orbit phase (SPO-1, March 27-April 28, 1998) has provided our first test of these algorithms. MOLA ranges at a 10-Hz, sampling rate with ~130 m diameter footprints spaced ~330 m along track with a noise level which is below the 37.5 cm range resolution of the altimeter time interval unit. The high eccentricity of the MGS orbit during SPO-1 [*Albee,* 1998] also contributes to the sensitivity of the altimetry from this period to horizontal orbit error.

Our crossover constraint equations have been formulated in terms of the minimum distance between two curves that have been traced out by the altimeter on the planet surface instead of a height-only discrepancy at a predetermined point. This approach requires that each crossover constraint equation takes into account a whole series of altimeter ranges from each of the two altimeter passes (ascending and descending) that surround the location where a conventional (height discrepancy) crossover occurs. These ranges are "geolocated", *i.e.,* the planet-fixed coordinates of the bounce points are determined by using knowledge of the spacecraft orbit and instrument pointing. For each pair of nearly intersecting passes we determine the two planet-fixed locations (and therefore the times) at which the passes come closest to intersecting. The distance between these two points represents the crossover discrepancy.

In order to describe how these crossover constraints interact in an orbit solution, it is important to point out that orbit determination is an iterative procedure. On each iteration, as the estimate of the orbit evolves, we re-determine the planet-fixed locations (and equivalently the times) at which the pairs of passes of MOLA altimetry come closest to intersecting. This re-determination is necessary since MOLA is returning data with high along track resolution over sloping terrain. In fact, the sloping terrain is taken into account in our crossover constraint equations through the use of short passes of altimetry (as opposed to single points). We represent the curves that are traced out by these passes as three dimensional polynomials. These curves are given their shape and orientation by topography and also by orbit and attitude parameters. In order to minimize the minimum distance between pairs of curves, it is necessary to match (or line up) the curves along the ascending and descending passes. To do this, it is often necessary to change the horizontal as well as the vertical orientation of these curves. In other words, our crossover minimization process directly affects all

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Paper number 1999GL900223.
0094-8276/99/1999GL900223$05.00

components of the orbit. When used with laser altimetry over sloping terrain, the form of crossover constraints described above has the potential to significantly improve the cross track component of the orbit, not just the radial and along track components.

Another facet of our crossover modeling is that we do not assume that the altimeter is nadir pointing. Laser altimeters typically operate at off-nadir angles. As a result we need to use spacecraft attitude and laser pointing information as part of the crossover computation (*i.e.* in the geolocation of bounce points). Thus the crossover constraint equations can also contribute to the refinement of spacecraft attitude and laser pointing parameters. With radar altimetry, pointing information can be refined by analyzing the waveform of the return pulse [*e.g. Hayne et al.*, 1994].

The estimation of attitude parameters from crossovers illustrates a point about the way we have chosen to write our constraint equations. When a satellite is pointing close to nadir, the direct effect on geolocation from error in attitude parameters is mainly horizontal. Standard crossover constraints are are written to minimize the height discrepancy at a fixed location. They offer no means of changing the location of the crossover as a way to minimize the height discrepancy. So, there is no practical way to estimate attitude parameters from standard crossovers when the satellite is fairly close to nadir pointing. This is not a problem when the crossover discrepancy to be minimized is the distance between two curves. Our crossover locations change after the orbit solution not just because the dynamics of the orbit have moved them. The crossover minimization process itself moves them. This also frees us from the worry that height discrepancies caused by horizontal orbit error will be aliased into the radial component. That could happen if crossover constraints are written as height discrepancies that are to be minimized at fixed locations.

Formulation of Laser Crossover Constraint Equations

In a least squares batch estimator, a constraint equation can be treated like an observation equation. In order to sum an observation into a set of normal equations, all that is needed is a residual (observation minus a computed observation), a weighting factor, and the partial derivatives of the computed observation with respect to all of the adjusting parameters. For crossovers, we let the observed value (i.e. the value to be fit by the computed) equal zero. The computed value is the crossover discrepancy and so the residual is the negative of the minimum distance. The weighting factor is the square of the reciprocal of a user-assigned crossover standard deviation. This standard deviation corresponds roughly to the expected crossover discrepancy after adjustment. For MGS we have used a value of five meters for the standard deviation of each crossover constraint equation. In order to explain how the partial derivatives are computed, it is necessary to provide additional details about the procedure for computing crossover discrepancies.

The geolocation process starts with satellite coordinates and pointing vectors at the time that the pulse is transmitted. A rigorous ray path from transmit location back to satellite at receive time is reconstructed in the inertial Mars J2000 IAU coordinate system [Davies et al., 1996]. This is the coordinate system in which the spacecraft ephemeris is computed and to which the spacecraft body fixed coordinate frame is related by telemetered quaternions. The inertial bounce point coordinates at the (reconstructed) bounce point time are converted to planet fixed using IAU parameters [Davis et al., 1996]. At this point, time varying corrections such as tides can be considered so that the bounce point can be referenced to a mean surface. The solid tides on Mars should be smaller than those on Earth (well under one meter) and we have neglected them in this study.

Once every range in a MOLA pass has been geolocated, the *X*, *Y* and *Z* planet-fixed coordinates of the bounce points can be fit to three polynomials in time. These polynomials describe a curve in space which can be compared to the curve (consisting of three more polynomials) on the other side of the crossover. The six polynomials are used to write a distance function. Given a pair of times (one from each pass), the distance between the passes at the associated points can be found. This function in two variables is easily minimized. The times that correspond to the minimum distance are found and this gives the crossover discrepancy.

We have not experimented extensively with the choice of polynomial degree and with the number of points used to fit the polynomial, although these are flexible options in our software. We have found that a quadratic fit by five points seems to work well. We do not use any crossover where any of the center three points are fit by one of the two quadratics worse than 5 meters. This is necessary since the polynomials are used to represent the discrete bounce points in the computation of minimum distance.

Using polynomials enables the computation of minimum distance and allows the computation of partial derivatives of minimum distance with respect to adjusted parameters to take surrounding topography into account. The minimum distance between two curves that have each been represented as three polynomials is a function of each of the coefficients of the six polynomials. These coefficients contain information about the topography along the ascending and descending passes. Embedded in these coefficients is the information about how change in crossover location relates to change in minimum distance. These coefficients are used to relate the adjusting parameters to change in minimum distance.

Orbit and Attitude Improvement for MGS Using Crossovers

The MGS Science Phasing Orbit-1 was a near-polar (93.7° inclination) orbit that encompassed the period of time from late March through April of 1998 [*Albee,* 1998]. All of the SPO-1 orbit solutions described in this section rely on ground based tracking including two-way ramped range [*Moyer,* 1995], two-way and three-way ramped Doppler [*Moyer,* 1987], and 1-way Doppler [*Moyer,* 1987]. We used the GEODYN orbit determination and geodetic parameter estimation software [*Pavlis et al.,* 1998] for our orbit solutions. We demonstrate below that the addition of crossover constraint equations as described in the above sections improves these orbit solutions. We also demonstrate that when knowledge of instrument pointing has been refined using crossovers, the orbit solutions are further improved.

The SPO-1 period was subdivided into six orbit solutions (arcs) each covering a little over six days. The start and stop times of each arc were chosen so that adjacent arcs would overlap by twelve hours, which is just larger than the orbital period for MGS during SPO-1 (11 hr 38 min). We will refer to three distinct solutions for these six arcs: SX0, SX5 and

Table 1. Orbit Overlap Comparisons

	Radial (m)			Cross-Track (m)			Along-Track (m)			Total (m)		
Overlap	SX0	SX5	SX5A	SX0	SX5	SX5A	SX0	SX5	SX5A	SX0	SX5	SX5A
3/27 - 4/01	11.7	16.3	13.8	738	107	91	27	45	36	785	117	99
4/01 - 4/07	4.6	11.1	8.9	1385	413	419	623	179	184	1519	450	458
4/07 - 4/12	1.9	13.0	10.6	362	165	208	197	82	106	413	185	233
4/12 - 4/18	30.7	17.2	16.1	419	635	560	262	416	367	495	759	670
4/18 - 4/24	24.7	24.6	24.6	1158	496	251	883	326	146	1457	594	292
RMS	18.5	17.1	15.8	907	414	347	505	253	201	1045	486	402

SX5A. These three solutions use the same data (ground tracking and altimetry) and solve for the same set of parameters. SX0 differs from SX5 only in the weight assigned to the 535 crossover constraint equations. In SX5, the standard deviation for the crossover constraints was set at five meters. In SX0 the standard deviation for crossovers was set at one million meters (*i.e.* no practical contribution). SX5A differs from SX5 only in the values assigned to certain pointing parameters. In none of the three SX solutions was any type of pointing parameter adjusted. Three types of attitude information were used in all three solutions: telemetered quaternions that describe the spacecraft orientation, a time tag bias for the quaternions and constant roll, pitch and yaw offset parameters that describe the offset in orientation of the MOLA instrument to the spacecraft body. The sense of the offset parameters is that a vector in the spacecraft body fixed frame is rotated to the MOLA instrument frame by first applying a roll rotation then a pitch rotation then a yaw rotation. The preflight values of the parameters are 0.021° for roll, -0.005° for pitch, and 0.059° for yaw.

Solution SX5A differs from SX5 only in the values for roll and pitch instrument pointing offset that were used. SX5 uses the preflight values. SX5A uses values for these that were adjusted from crossovers in a preliminary solution that we describe next. Other preliminary solutions for orbits also solved for a (MOLA) observation timing bias. From these earlier orbit solutions we have adopted a value of 0.114 seconds. All of the SX solutions used this value.

We produced separate solutions for attitude and orbit parameters because the crossovers that contribute most to attitude information (where the spacecraft is pointing well off nadir for either the ascending or descending pass) are the least desirable to use for orbit improvement. We developed a solution for attitude parameters using only 279 of the most off-nadir crossovers. In this solution, crossovers were allowed to contribute only to the solution of attitude parameters. The adjusted parameters were a telemetered spacecraft attitude timing bias and a constant offset in roll and pitch.

It should be pointed out that we are directly solving for the pointing of of the MOLA instrument. This is not separable from the overall orientation of the MGS spacecraft. However, in the case of the telemetered attitude timing bias, the refinement in attitude should apply to the entire spacecraft body (including antennae) and results in attitude refinement which is by no means a constant offset.

It was necessary to solve for an attitude timing bias. On MGS this bias probably results from delays in recording the attitude sensor data. The bias can be seen during rapid rollouts from nadir pointing. Geolocated altimetry passes during these periods show a characteristic dip which can be removed by manipulating the attitude time tag. Our adjusted value is 1.160 seconds, and even the discrepancies of crossovers from "quiet" passes improve with the application of this bias. By comparing the results of SX5 and SX5A (below) it can be seen that our adjusted value of roll and pitch improve the orbit solution (Table 1). Although this improvement is modest, it is an independent means of confirming that our attitude adjustment is beneficial. The adjustment of attitude does have a very significant effect on geolocation (Table 3).

We gauge orbit quality in three ways: by looking at orbit overlap statistics, formal standard deviations of adjusted parameters and crossover discrepancies.

Table 1 shows the five 12 hour overlaps between the six arcs in SPO-1 for the three SX solutions. The table shows that inclusion of MOLA altimetry improves total positioning of the satellite, mainly through horizontal improvement. The improvement is slightly better if MOLA observations are allowed to contribute pointing information. In evaluating the improvements it is important to note that MGS was in a highly eccentric orbit during SPO-1 and altimetry observations could only be made for approximately one half hour of each 11-hour 38-minute revolution. It is also worth noting that the altimeter only returned data during periapsis when the spacecraft was usually well-tracked from Earth. As a result, overlap discrepancies during these periapsis pass portions of the orbit tend to be smaller than elsewhere,

Table 2. Kepler Epoch State Vector Recovered Sigmas

	A (m)		e (10^{-8})		I (10^{-5} deg)		Ω (10^{-5} deg)		ω (10^{-5} deg)		M (10^{-6} deg)	
Arc	SX0	SX5	SX0	SX5	SX0	SX5	SX0	SX5	SX0	SX5	SX0	SX5
3/27	14.3	14.2	34	33	117	47	47	24	30	10	365	362
4/01	0.6	0.6	23	20	92	37	38	15	31	12	19	16
4/07	2.4	2.1	11	8	60	31	22	11	25	13	80	65
4/12	2.5	2.2	13	9	103	36	38	13	52	18	44	35
4/18	5.6	1.1	17	9	55	27	19	10	32	16	31	13
4/24	3.5	3.1	17	9	38	31	12	10	27	22	97	91
RMS	16.1	14.9	51	42	202	87	78	36	83	38	390	381

Table 3. Crossover Discrepancy

Pointing offset	RMS (m)		
	SX0	SX5	SX5A
Pre-flight	26.08	15.67	15.26
Adjusted	22.87	9.40	9.37

whether or not altimetry is included. The inclusion of MOLA data should ultimately be even more useful when it can be applied over the entire orbit -- after the MGS orbit is circularized at the end of aerobraking [*Albee,* 1998].

Table 2 gives the formal standard deviation of the six initial kepler state parameters of the six arcs. As would be expected, the standard deviations are improved by the addition of constraint equations. However, it is interesting to note that the parameters that seem to be generally the most improved are inclination *(I)*, right ascension of the node *(Ω)* and the argument of perigee *(ω)*. These correspond to cross track (*I* and *Ω*) and along track *(ω)* components of the orbit and this correlates well with the results shown in Table I. There is only one arc (5) for which the standard deviation of the semi-major axis *(A)* is dramatically improved. This corresponds to an improvement in the radial component and seems to correlate well with the only overlap for which there is a dramatic improvement in the radial component (overlap 4/12-4/18 between arcs 4 and 5). The agreement between Tables 1 and 2 is encouraging.

Table 3 gives the Root Mean Square (RMS) discrepancies of the 1180 SPO-1 crossovers that occur under circumstances that are considered suitable for the computation of crossover discrepancies. These are presented for the three SX orbit solutions. In all cases the telemetered attitude timing bias and the observation timing bias were applied. For each orbit solution the crossovers are presented with the pointing done two ways: with the preflight values of roll and pitch offsets and with the adjusted values. Either way there is improvement from SX0 to SX5 and from SX5 to SX5A. For each orbit solution there is improvement when the adjusted values of pointing are used. Table 3 further supports the claim that the crossovers have improved the orbits and that pointing adjustment has improved both the orbit and the geolocation.

Conclusions

Satellite laser altimetry in the form of crossover constraint equations has been used to significantly improve the spacecraft orbit and attitude solutions of MGS during SPO-1. The small footprint and associated high along track resolution have been exploited using "minimum distance" crossover constraints to improve all components of the satellite orbit, most notably the cross track component. The resulting orbits used for geolocation during SPO-1 have a radial precision of better than 25 m RMS. The geolocation of MOLA bounce points has been improved by both the orbit and the attitude solutions and by the solution of an altimeter observation timing bias. The values for these geolocation parameters are as follows: 0.114 seconds for observation timing bias, 1.160 seconds for telemetered attitude timing bias, –0.0028° for roll offset and –0.0086° for pitch offset.

The minimum distance technique described in this paper is not limited in use to the MGS mission. It should be useful in future laser altimeter missions (NEAR, VCL, ICESAT and SELENE).

Acknowledgments. The authors are grateful to Theodore Moyer of JPL for his helpful advice on the implementation of 1-way interplanetary Doppler data into GEODYN. We would like to thank George Hayne and David Hancock of the Wallops Flight Facility of NASA Goddard for some useful discussions about radar altimetry. Jeffery Schott of MIT and Susan Fricke of Raytheon ITSS provided technical support. Richard Ray and Maria Zuber provided useful criticism at on site reviews. Steve Nerem and an anonymous reviewer provided useful criticisms at GRL. This work was supported by the NASA Mars Surveyor Project under a grant to D.E. Smith.

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(Received January 22, 1999; revised March 15, 1999;
accepted March 22, 1999.)